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Supersymmetric new brane-world

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ABSTRACT

The quantum-induced dilatonic brane world (New Brane World) is created by brane CFT quantum effects (giving effective brane tension) in accordance with AdS/CFT set-up which also defines surface term. Considering bosonic sector of 5d gauged supergravity with single scalar and taking the boundary action as predicted by supersymmetry, the possibility to supersymmetrize dilatonic New Brane World is discussed. It is demonstrated that for number of superpotentials the flat SUSY dilatonic brane-world (with dynamically induced brane dilaton) or quantum-induced de Sitter dilatonic brane-world (not Anti-de Sitter one) where SUSY is broken by quantum effects occurs. The analysis of graviton perturbations indicates that gravity is localized on such branes.

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1 Introduction

Recent increasing interest in the study of brane-worlds is caused by several reasons. First of all, it has been realized that 4d brane gravity may be trapped [1, 2]. Second, the natural proposal to resolve the mass hierarchy problem appears [1].

The interesting variant of brane-world scenario has been suggested in refs.[3, 4]. (It has been called New Brane World [4]). In this scenario, brane-worlds are naturally realized in frames of AdS/CFT duality [5]. Unlike to convenient brane-worlds, the boundary action is not the arbitrary one (brane tension is not free parameter). On the contrary, the surface terms on AdS-like space are motivated by AdS/CFT correspondence. Their role is in making of variational procedure to be well-defined and in the elimination of the leading divergence of the AdS-like action. In other words, brane tension is not free parameter but it is fixed by the condition of finiteness of spacetime when brane goes to infinity. However, in accordance with AdS/CFT correspondence there is quantum CFT living on the brane. Such brane quantum CFT produces conformal anomaly (or anomaly induced effective action) which leads to creation of effective brane tension. As a result the dynamical mechanism to get flat or curved (de Sitter or Anti-de Sitter) brane-world appears [3, 4] (for study of related questions in this scenario see [6]) in frames of AdS/CFT duality. Hence, one gets less fine-tuning in realization of brane-worlds as brane tension is not free parameter. The nice feature of this dynamical scenario is that sign of conformal anomaly terms for usual matter predicts de Sitter (inflationary) Universe as a preferrable solution in one-brane case.

The scenario of refs.[3, 4] may be extended to the presence of non-trivial dilaton as it was done in refs.[7]. Then, whole scenario looks even more related with AdS/CFT correspondence as dilatonic gravity naturally follows as bosonic sector of d5 gauged supergravity (with special parametrization of scalars space). The number of quantum-induced curved (de Sitter or Anti-de Sitter) dilatonic brane-worlds has been constructed in refs.[7].

From another side, there is much activity now in supersymmetrization of Randall-Sundrum brane world [8, 9, 10, 11] (see also refs. therein). 5d gauged supergravity represents very interesting model where supersymmetric dilatonic brane-world should be searched. Moreover, in such model it is natural to try to construct supersymmetric dilatonic brane-world consistent with AdS/CFT correspondence [5]. It could be then that such scenario should be realized as supersymmetric version of New Brane World [3, 4, 7]. In the present work we make an attempt in the construction of supersymmetric New Brane World.

In the next section the review of the construction of classical supersymmetric brane-world is done for bosonic sector of 5d gauged SG with single dilaton. Boundary action is predicted by supersymmetry. Half of supersymmetries survives for flat brane-world (as it follows from the analysis of BPS condition). The classical SUSY curved brane-worlds cannot be realized. Third section is devoted to the extension of the analysis of second section modified by the quantum contribution from brane CFT in order to construct SUSY New Brane-World. It is shown for number of superpotentials that unlike to classical case the quantum induced de Sitter brane-world is created. However, brane supersymmetry is broken by quantum effects. The example of SUSY flat brane-world where boundary value of dilaton is defined by quantum effects is also given. In section four the analysis of graviton perturbations around found solutions is done. It is shown that only one normalizable solution corresponding to zero mode exists. In other words, gravity should be trapped on the brane in such scenario. Some brief summary and outlook is given in final section.

2 Classical supersymmetric brane-world

The 5d $\mathcal{N}=8$ gauged supergravity can be obtained from 10d IIB supergravity, where the spacetime is compactified into $\mathrm{S}_5\times\mathrm{M}_5$, where S_5 is 5d sphere and M_5 is a 5d manifold, where the gauged supergravity lives. The bosonic sector (gravity and scalar part) of the 5d gauged supergravity is given by

$$S_{\rm bulk}=\frac{1}{16\pi G}\int d^5x\sqrt{g_{(5)}}\left(R_{(5)}-\frac{1}{2}g_{ij}(\phi_k)\nabla_\mu\phi^i\nabla^\mu\phi^j+V(\phi^i)\right)\ . \qquad (1)$$

Here $g_{ij}(\phi^k)$ is the induced scalars metric and the potential $V(\phi^i)$ is given in terms of the superpotential $W(\phi_i)$:

$$V(\phi_i)=-\frac{3}{4}\left(\frac{3}{2}g^{ij}(\phi_k)\frac{dW(\phi_k)}{d\phi^i}\frac{dW(\phi_k)}{d\phi^j}-W(\phi_k)^2\right)\ . \qquad (2)$$

When there are boundaries or branes in 5d spacetime, it has been shown in [10] that supersymmetry of the whole system consisting of the bulk and brane(s) is preserved by introducing a scalar field $\tilde{G}$ and four form field $A_{\mu\nu\rho\sigma}$ and adding the following action

$$S_A = \frac{1}{4!4\pi G}\int d^5x \epsilon^{\mu\nu\rho\sigma\tau} A_{\mu\nu\rho\sigma}\partial_\tau \tilde{G} \tag{3}$$

in the bulk spacetime and the boundary action

$$S_{\rm bndry} = \mp\frac{1}{8\pi G}\int d^4x \left(\frac{3}{2}W(\phi)\sqrt{g_{(4)}} + \frac{2g}{4!}\epsilon^{\mu\nu\rho\sigma}A_{\mu\nu\rho\sigma}\right) \tag{4}$$

to $S_{\rm bulk}$ (1). Here g is a gauge coupling constant. The sign $\mp$ comes from the ambiguity when we solve (2) with respect to $W(\phi)$ but if there are two branes at $z = 0$ and $z = R_b > 0$, the relative sign should be different. On shell, where $\tilde{G} = g{\rm sgn}(z)$[3], the boundary action $S_{\rm bndry}$ (4) has the following form:

$$S_{\rm bndry} = \mp\frac{3}{16\pi G}\int d^4x \sqrt{g_{(4)}}W(\phi)\ . \tag{5}$$

This tells that the brane is BPS saturated state, that is, the brane preserves the half of the supersymmetries of the whole system. Therefore if 5d supergraviry is $\mathcal{N} = 8$ gauged one, 4d $\mathcal{N} = 4$ super-Yang-Mills coupled with supergravity would be realized on the branes. In order to see it, one considers the simple case that only one scalar field ϕ is non-trivial and $g_{\phi\phi} = 1$ and investigate the equations of motion given by the simplified action:

$$\begin{aligned} S &= \frac{1}{16\pi G}\left[\int_M d^5x\sqrt{-g_{(5)}}\left(R_{(5)} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right)\right. \\ &\left.\quad - \sum_{i=1,2}\int_{B_i} d^4x\sqrt{-g_{(4)}}U_i(\phi)\right]\ . \end{aligned} \tag{6}$$

Here B_i's express the boundaries or branes. At first, we do not specify the form of $U_i(\phi)$ but by investigating the equations of motion, we will see the correspondence with (5). We now assume the metric in 5d spacetime as

$$ds^2 = dz^2 + {\rm e}^{2A(z)}\eta_{ij}dx^i dx^j\ , \tag{7}$$

[3]Here the function ${\rm sgn}(x)$ is defined by

$${\rm sgn}(x) = \left\{\begin{array}{ll} 1 & x > 0 \\ -1 & x < 0 \end{array}\right.$$

and z is the coordinate perpendicular to the boundary or brane.

and ϕ only depends on z. One also supposes the branes sit on $z = z_1$ and $z = z_2$, respectively. Then the equations of motion are given by

$$\phi'' + 4A'\phi' = -\frac{dV}{d\phi} + \sum_{i=1,2} \frac{dU_i(\phi)}{d\phi}\delta(z - z_i) \ , \tag{8}$$

$$\begin{aligned} & 4A'' + 4(A')^2 + \frac{1}{2}(\phi')^2 \\ & \quad = \frac{V}{3} - \frac{2}{3}\sum_{i=1,2} U_i(\phi)\delta(z - z_i) \ , \end{aligned} \tag{9}$$

$$A'' + 4(A')^2 = \frac{V}{3} - \frac{1}{6}\sum_{i=1,2} U_i(\phi)\delta(z - z_i) \ . \tag{10}$$

Here $' \equiv \frac{d}{dz}$. For purely bulk sector ($z_1 < z < z_2$, as we assume $z_1 < z_2$), Eqs. (8-10) have the following first integrals:

$$\phi' = \frac{3}{2}\frac{dW}{d\phi} \ , \quad A' = -\frac{1}{4}W \ . \tag{11}$$

(Here the ambiguity in the sign when solving Eq.(2) with respect to $W(\phi)$ is absorbed into the definition of $W(\phi)$.) One should note that classical solutions do not always satisfy the above Eqs. in (11). Classical solutions are generally not invariant under the supersymmetry transformations and the supersymmetry in the bulk is broken in the classical background. Eq.(11) is nothing but the condition that the classical solution is invariant under the half of the supersymmetry transformations. When there are branes, any solution of the equations of motion including the equation coming from the branes might not satisfy Eqs.(11). We now investigate the condition for the brane action which allows solution satisfying Eqs.(11). Then some of the supersymmetries are preserved in the whole system.

Near the branes, Eqs. (8-10) have the following form :

$$\phi'' \sim \frac{dU_i(\phi)}{d\phi}\delta(z - z_i) \ , \quad A'' \sim -\frac{U_i(\phi)}{6}\delta(z - z_i) \ , \tag{12}$$

or

$$2\phi' \sim \frac{dU_i(\phi)}{d\phi}, \quad 2A' \sim -\frac{U_i(\phi)}{6} \ , \tag{13}$$

at $z = z_i$. If Eqs.(11) are satisfied, one finds

$$U_1(\phi) = 3W(\phi) \ , \quad U_2(\phi) = -3W(\phi) \ . \tag{14}$$

Eq.(14) reproduces Eq.(5). Note that Eq.(11) is nothing but the BPS condition, where the half of the supersymmetries in the whole system are preserved. As we are considering the solution where fermionic fields vanish, the variations of the fermionic fields under the supersymmetry transformation should vanish if the solution preserves the supersymmetry. If Eq.(11) is satisfied, the variations of gravitino and dilatino vanish under the half of the supersymmetry transformation.

As an extension, one can consider the case that the brane is curved. Instead of (7), we take the following metric:

$$ds^2 = dz^2 + \mathrm{e}^{2A(z)}\tilde{g}_{ij}dx^i dx^j \ , \tag{15}$$

Here $\tilde{g}_{ij}$ is the metric of the Einstein manifold, which is defined by

$$\tilde{R}_{ij} = k\tilde{g}_{ij} \ , \tag{16}$$

where $\tilde{R}_{ij}$ is the Ricci tensor given by $\tilde{g}_{ij}$ and k is a constant. Then Eqs.(8) and (9) do no change but one obtains the following equation instead of (10):

$$A'' + 4(A')^2 = k\mathrm{e}^{2A} - \frac{V}{3} - \frac{1}{6}\sum_{i=1,2} U_i(\phi)\delta(z - z_i) \ . \tag{17}$$

Especialy when $k = 0$, one gets the previous solution for ϕ, A and U_i. Even for $k = 0$, the brane is not always flat, for example, if as $\tilde{g}_{ij}$ in (15), we choose the metric of the Schwarzschild black hole or Kerr black hole spacetime, then Eq.(16) is satisfied since the Ricci tensor vanishes.

Therefore the brane solutions with these black holes of $k = 0$ would preserve the supersymmetry of the whole system. When $k \neq 0$, however, one finds that Eq.(17) has no solution which satisfies the BPS condition (11). This might tell that classical curved brane breaks the supersymmetry in such formalism. When $k > 0$, the brane is 4d de Sitter space or 4d sphere when Wick-rotated to the Euclidean signature. On the other hand, when $k < 0$, the brane is 4d anti-de Sitter space or 4d hyperboloid in the Euclidean signature.

3 Supersymmetric new brane world

In the previous section, the discussion was mainly limited by flat brane. In this case, however, the brane crosses the event horizon in the finite time, which opens the causality problem. To avoid this problem, it would be natural to consider de Sitter brane which is also motivated by cosmology. If the brane is de Sitter space, the brane does not cross the horizon. Motivated by this, we consider the curved brane in this section, although the classical curved brane seems to break supersymmetry in general, as it was seen in the previous section.

If 10d spacetime, where IIB supergraviry lives, is compactified into $\mathrm{S}_5\times\mathrm{M}_5$, we effectively obtain 5d $\mathcal{N}=8$ gauged supergravity in the bulk and 4d $\mathcal{N}=4$ $SU(N)$ or $U(N)$ super-Yang-Mills theory coupled with (super)gravity on the brane. On the other hand, if 10d spacetime is compactified into $\mathrm{X}_5\times\mathrm{M}_5$, where X_5 is S_5/Z_2, $\mathcal{N}=2$ $Sp(N)$ super-Yang-Mills theory coupled with (super)gravity would be realized on the brane. Since the matter multiplets of the super-Yang-Mills are coupled with (super)gravity, they generate a conformal anomaly on quantum level. In [4, 3], it has been shown that the curved brane, which is 4d de Sitter space, can be generated by including the conformal anomaly (via account of corresponding effective tension). In this way, quantum induced brane-worlds appear in frames of AdS/CFT duality[5] as it has been explained in refs.[4, 3]. Only non-supersymmetric configurations (for generalization on the non-constant dilaton presence, see [7]) have been considered.

In this section, we include the trace anomaly induced action on the brane to the analysis of supersymmetric brane-world. One chooses the brane action to preserve the supersymmetry as in the previous section and consider the solution where the scalar field is non-trivial. Here mainly Euclidean signature is used.

As curved brane is considered, we assume that the metric of (Euclidean) AdS has the following form:

$$ds^2 = dz^2 + \sum_{i,j=1}^{4} g_{(4)ij}dx^i dx^j \ , \quad g_{(4)ij} = \mathrm{e}^{2\tilde{A}(z)}\hat{g}_{ij} \ . \tag{18}$$

Here $\hat{g}_{ij}$ is the metric of the Einstein manifold as in (15). One can consider two copies of the regions given by $z < z_0$ and glue two regions putting a brane at $z = z_0$.

Let us start with Euclidean signature action S which is the sum of the Einstein-Hilbert action $S_{\rm EH}$ with kinetic term and potential $V(\phi)$ for dilaton ϕ, the Gibbons-Hawking surface term $S_{\rm GH}$, the surface counter term S_1 and the trace anomaly induced action $\mathcal{W}$ [7]:

$$\begin{aligned}
S &= S_{\rm EH} + S_{\rm GH} + 2S_1 + \mathcal{W}, && (19)\\
S_{\rm EH} &= \frac{1}{16\pi G}\int d^5x\sqrt{g_{(5)}}\left(R_{(5)} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V(\phi)\right), && (20)\\
S_{\rm GH} &= \frac{1}{8\pi G}\int d^4x\sqrt{g_{(4)}}\nabla_\mu n^\mu, && (21)\\
S_1 &= -\frac{3}{16\pi G l}\int d^4x\sqrt{g_{(4)}}W(\phi), && (22)\\
\mathcal{W} &= b\int d^4x\sqrt{\widetilde{g}}\widetilde{F}A \\
&\quad +b'\int d^4x\sqrt{\widetilde{g}}\left\{A\left[2\widetilde{\Box}^2 + \widetilde{R}_{\mu\nu}\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu - \frac{4}{3}\widetilde{R}\widetilde{\Box}^2 + \frac{2}{3}(\widetilde{\nabla}^\mu\widetilde{R})\widetilde{\nabla}_\mu\right]A\right. \\
&\quad \left. + \left(\widetilde{G} - \frac{2}{3}\widetilde{\Box}\widetilde{R}\right)A\right\} && (23)\\
&\quad -\frac{1}{12}\left\{b'' + \frac{2}{3}(b+b')\right\}\int d^4x\sqrt{\widetilde{g}}\left[\widetilde{R} - 6\widetilde{\Box}A - 6(\widetilde{\nabla}_\mu A)(\widetilde{\nabla}^\mu A)\right]^2 \\
&\quad +C\int d^4x\sqrt{\widetilde{g}}A\phi\left[\widetilde{\Box}^2 + 2\widetilde{R}_{\mu\nu}\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu - \frac{2}{3}\widetilde{R}\widetilde{\Box}^2 + \frac{1}{3}(\widetilde{\nabla}^\mu\widetilde{R})\widetilde{\nabla}_\mu\right]\phi\ .
\end{aligned}$$

Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices $_{(5)}$ and those in the boundary 4 dimensional spacetime are specified by $_{(4)}$.

In [7], as an action on the brane, corresponding to S_1 in (19), the action motivated by the counterterm method in AdS/CFT correspondence was used:

$$S_1^{\rm NOO} = -\frac{1}{16\pi G l}\int d^4x\sqrt{g_{(4)}}\left(\frac{6}{l} + \frac{l}{4}\Phi(\phi)\right)\ . \qquad (24)$$

In the AdS/CFT correspondence, the divergence coming from the infinite volume of AdS corresponds to the UV divergence in the CFT side. The counterterm which cancells the leading divergence in the AdS side corresponds to the above action $S_1^{\rm NOO}$.

In the present framework, the spacetime inside the brane has finite volume and there might be ambiguities when choosing the counterterm. Here we give

S_1 in (19) in terms of the superpotential $W(\phi)$ corresponding to (2) which is given by

$$V(\phi) = -\frac{3}{4}\left(\frac{3}{2}\left(\frac{dW(\phi)}{d\phi}\right)^2 - W(\phi)^2\right) \ . \tag{25}$$

This is natural in terms of supersymmetric extension [11] of the Randall-Sundrum model [1, 2]. This action tells that the brane is BPS saturated state and the half of the supersymmetries could be conserved [10]. The factor 2 in front of S_1 in (19) is coming from that we have two bulk regions which are connected with each other by the brane.

In (21), n^μ is the unit vector normal to the boundary. In (21), (22) and (23), one chooses the 4 dimensional boundary metric as

$$g_{(4)\mu\nu} = \mathrm{e}^{2A}\tilde{g}_{\mu\nu}, \tag{26}$$

We should distinguish A and $\tilde{g}_{\mu\nu}$ with $\tilde{A}(z)$ and $\hat{g}_{ij}$ in (18). We will specify $\hat{g}_{ij}$ later in (40). We also specify the quantities given by $\tilde{g}_{\mu\nu}$ by using $\tilde{\ }$.

In (23), G ($\tilde{G}$) and F ($\tilde{F}$) are the Gauss-Bonnet invariant and the square of the Weyl tensor, which are given as

$$\begin{aligned} G &= R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl}, \\ F &= \frac{1}{3}R^2 - 2R_{ij}R^{ij} + R_{ijkl}R^{ijkl} \ , \end{aligned} \tag{27}$$

In the effective action (23) induced by brane quantum matter, in general, with N scalar, $N_{1/2}$ spinor, N_1 vector fields, N_2 ($= 0$ or 1) gravitons and N_{HD} higher derivative conformal scalars, b, b' and b'' are

$$\begin{aligned} b &= \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\mathrm{HD}}}{120(4\pi)^2} \\ b' &= -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\mathrm{HD}}}{360(4\pi)^2} \ , \\ b'' &= 0 \ . \end{aligned} \tag{28}$$

Usually, b'' may be changed by the finite renormalization of local counterterm in gravitational effective action. As it was the case in ref.[7], the term proportional to $\left\{b'' + \frac{2}{3}(b + b')\right\}$ in (23), and therefore b'', does not contribute to the equations of motion. Note that CFT matter induced effective action may be considered as brane dilatonic gravity.

For typical examples motivated by AdS/CFT correspondence[5] one has:
a) $\mathcal{N}=4$ $SU(N)$ SYM theory

$$b=-b'=\frac{C}{4}=\frac{N^2-1}{4(4\pi)^2}\ , \tag{29}$$

b) $\mathcal{N}=2$ $Sp(N)$ theory

$$b=\frac{12N^2+18N-2}{24(4\pi)^2}\ , \quad b'=-\frac{12N^2+12N-1}{24(4\pi)^2}\ . \tag{30}$$

We should note that b' is negative in the above cases.

Let us start the consideration of field equations. It is often convienient that one assumes the metric of 5 dimensional spacetime as follows:

$$ds^2=g_{(5)\mu\nu}dx^\mu dx^\nu=f(y)dy^2+y\sum_{i,j=1}^{4}\hat{g}_{ij}(x^k)dx^i dx^j. \tag{31}$$

Here $\hat{g}_{ij}$ is the metric of the 4 dimensional Einstein manifold as in (18). A coordinate corresponding to z in (18) can be obtained by

$$z=\int dy\sqrt{f(y)}, \tag{32}$$

and solves y with respect to z. Then the warp factor is $\mathrm{e}^{2\hat{A}(z,k)}=y(z)$.

From the variation over $g_{(5)\mu\nu}$ in the Einstein-Hilbert action (20), we obtain the following equation in the bulk

$$\begin{aligned}0 &= R_{(5)\mu\nu}-\frac{1}{2}g_{(5)\mu\nu}R-\frac{1}{2}V(\phi)g_{(5)\mu\nu} \\ &\quad -\frac{1}{2}\left(\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g_{(5)\mu\nu}g_{(5)}^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi\right)\end{aligned} \tag{33}$$

and from that over dilaton ϕ

$$0=\partial_\mu\left(\sqrt{g_{(5)}}g_{(5)}^{\mu\nu}\partial_\nu\phi\right)+\frac{dV(\phi)}{d\phi}\ . \tag{34}$$

Assuming that $g_{(5)\mu\nu}$ is given by (31) and ϕ depends only on y: $\phi = \phi(y)$, we find the equations of motion (33) and (34) take the following forms:

$$\begin{aligned} 0 &= \frac{2kf}{y} - \frac{3}{2}\frac{1}{y^2} + \frac{1}{2}V(\phi)f + \frac{1}{4}\left(\frac{d\phi}{dy}\right)^2 && (35) \\ 0 &= \frac{kf}{y} + \frac{3}{4fy}\frac{df}{dy} + \frac{1}{2}V(\phi)f - \frac{1}{4}\left(\frac{d\phi}{dy}\right)^2 && (36) \\ 0 &= \frac{d}{dy}\left(\frac{y^2}{\sqrt{f}}\frac{d\phi}{dy}\right) + \frac{dV(\phi)}{d\phi}y^2\sqrt{f}\ . && (37) \end{aligned}$$

Eq.(35) corresponds to $(\mu,\nu) = (y,y)$ in (33) and Eq.(36) to $(\mu,\nu) = (i,j)$. The case of $(\mu,\nu) = (y,i)$ or (i,y) is identically satisfied.

On the other hand, brane equations are

$$\begin{aligned} 0 &= \frac{48l^4}{16\pi G}\left(\partial_z A - \frac{1}{2}W(\phi)\right)\mathrm{e}^{4A} + b'\left(4\partial_\sigma^4 A - 16\partial_\sigma^2 A\right) \\ &\quad -4(b+b')\left(\partial_\sigma^4 A + 2\partial_\sigma^2 A - 6(\partial_\sigma A)^2\partial_\sigma^2 A\right) \\ &\quad +2C\left(\partial_\sigma^4\phi - 4\partial_\sigma^2\phi\right), && (38) \\ 0 &= -\frac{l^4}{8\pi G}\mathrm{e}^{4A}\partial_z\phi - \frac{3l^3\mathrm{e}^{4A}}{8\pi G}\frac{dW(\phi)}{d\phi} \\ &\quad +C\left\{A\left(\partial_\sigma^4\phi - 4\partial_\sigma^2\phi\right) + \partial_\sigma^4(A\phi) - 4\partial_\sigma^2(A\phi)\right\}\ . && (39) \end{aligned}$$

In (38) and (39), using the coordinate z in (32) and choosing $l^2\mathrm{e}^{2\hat{A}(z,k)} = y(z)$ one uses the form of the metric as

$$ds^2 = dz^2 + \mathrm{e}^{2A(z,\sigma)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu\ , \quad \tilde{g}_{\mu\nu}dx^\mu dx^\nu \equiv l^2\left(d\sigma^2 + d\Omega_3^2\right)\ . \qquad (40)$$

Here $d\Omega_3^2$ corresponds to the metric of 3 dimensional unit sphere. Then for the unit sphere ($k = 3$)

$$A(z,\sigma) = \hat{A}(z, k=3) - \ln\cosh\sigma\ , \qquad (41)$$

for the flat Euclidean space ($k = 0$)

$$A(z,\sigma) = \hat{A}(z, k=0) + \sigma\ , \qquad (42)$$

and for the unit hyperboloid ($k=-3$)

$$A(z,\sigma) = \hat{A}(z, k=-3) - \ln\sinh\sigma \ . \tag{43}$$

We now identify A and $\tilde{g}$ in (40) with those in (26). Then one finds $\tilde{F} = \tilde{G} = 0$, $\tilde{R} = \frac{6}{l^2}$ etc. Note that sphere in (41) corresponds to de Sitter space and the hyperboloid in (43) to anti-de Sitter space when we Wick-rotate the Euclidean signature to the Lorentzian one.

Using (35) and (37), one can delete f from the equations and can obtain an equation that contains only the dilaton field ϕ (and, of course, bulk potential):

$$\begin{aligned} 0 \ &= \ \left\{ \frac{5k}{2} - \frac{k}{4}y^2 \left(\frac{d\phi}{dy}\right)^2 + \left(\frac{3}{2}y - \frac{y^3}{6}\left(\frac{d\phi}{dy}\right)^2\right)\frac{V(\phi)}{2}\right\}\frac{d\phi}{dy} \\ &\quad + \frac{y^2}{2}\left(\frac{2k}{y} + \frac{1}{2}V(\phi)\right)\frac{d^2\phi}{dy^2} + \left(\frac{3}{4} - \frac{y^2}{8}\left(\frac{d\phi}{dy}\right)^2\right)\frac{dV(\phi)}{d\phi} \ . \end{aligned} \tag{44}$$

Several solutions have been found in second ref.[7] by assuming the dilaton and bulk potentials as:

$$\begin{aligned} \phi(y) \ &= \ p_1 \ln(p_2 y) \\ V(\phi) \ &= \ c_1 \exp(a\phi) + c_2 \exp(2a\phi) \ , \end{aligned} \tag{45, 46}$$

where a, p_1, p_2, c_1, c_2 are some constants. When $p_1 = \pm\frac{1}{\sqrt{6}}$, Eq.(44) is always satisfied but from Eq.(37) one gets that $f(y)$ identically vanishes. Therefore natural restriction is $p_1 \neq \pm\frac{1}{\sqrt{6}}$.

When $k \neq 0$, a special solution is given by

$$\begin{aligned} & c_1 = \frac{6kp_2p_1^2}{3-2p_1^2} \ , \quad c_2 = 0 \ , \quad a = -\frac{1}{p_1} \ , \quad p_1 \neq \pm\sqrt{6} \\ & f(y) = \frac{3-2p_1^2}{4ky} \ . \end{aligned} \tag{47}$$

One can check that above solution satisfies (36). Here the superpotential $W(\phi)$ is given by

$$W(\phi) = 8p_1^2\sqrt{\frac{p_2 k}{(3-2p_1^2)(8p_1^2-3)}}\mathrm{e}^{-\frac{\phi}{2p_1}} \ . \tag{48}$$

The potential (46) with the coefficients c_1 and c_2 in (47) corresponds to special RG flow in 5d $\mathcal{N}=8$ gauged supergravity where only one scalar from 42 scalars is considered. If we define q^2 by

$$q^2 \equiv \frac{4k}{3-2p_1^2} > 0 \ , \tag{49}$$

the solution when $k=0$ can be obtained by taking $k\to 0$ limit and keeping q^2 finite. In the limit, Eqs.(47) and (48) have the folowing forms:

$$c_1 = \frac{9}{4}q^2 p_2 \ , \quad c_2 = 0 \ , \quad a = -\frac{1}{p_1} \ , \quad p_1 \neq \pm\sqrt{6} \ , \quad f(y) = \frac{1}{q^2 y} \tag{50}$$

$$W(\phi) = 2\sqrt{q^2 p_2}\,\mathrm{e}^{-\phi\sqrt{\frac{3}{2}}} \ . \tag{51}$$

This solution satisfies Eq.(11), which is the BPS condition, i.e., the solution preserves the half of the supersymmetries in the bulk space.

The solutions in (47) and (50) have a singularity at $y=0$. In fact the scalar curvature $R_{(5)}$ is given by

$$R_{(5)} = -\frac{3}{2}\frac{p_1^2 q^2}{y} \ . \tag{52}$$

Here we assume q^2 is defined by (49) even if $k\neq 0$. When $k=3$, the brane becomes de Sitter space after the Wick-rotation. Then $y=0$ corresponds to the horizon in the bulk 5d space. Therefore in $k=3$ case, the singularity is not exactly naked.

In the coordinate system (31), brane Eq.(39) has the following form:

$$0 = -\frac{y_0^2}{8\pi G\sqrt{f(y_0)}}\partial_y\phi - \frac{3y_0^2}{8\pi G}\frac{dW(\phi_0)}{d\phi} + 6C\phi_0 \ . \tag{53}$$

Here ϕ_0 ($\tilde{\phi}_0$) is the value of the dilaton ϕ on the brane. We also find Eq.(38) has the following form:

$$0 = \frac{3y_0^2}{16\pi G}\left(\frac{1}{2y_0\sqrt{f(y_0)}} - \frac{l}{2}W(\phi_0)\right) + 8b' \tag{54}$$

for $k\neq 0$ case and

$$0 = \frac{3y_0^2}{16\pi G}\left(\frac{1}{2y_0\sqrt{f(y_0)}} - \frac{l}{2}W(\phi_0)\right) \tag{55}$$

for $k = 0$ case.

When $k = 0$, where $p_1^2 = \frac{3}{2}$, Eq.(55) is satisfied trivially but Eq.(57) has the following form:

$$-\frac{qy_0^{\frac{3}{2}}}{8\pi G}\sqrt{\frac{3}{2}} = 6C\phi_0 \ . \tag{56}$$

Then the value ϕ_0 of the dilaton on the brane depends on y_0. We should note that the obtained solution for $k = 0$ is really supersymmetric in the whole system since the corresponding bulk solution (50) satisfies the BPS condition Eq.(11) which tells the solution preserves the half of the supersymmetries in the bulk space and the brane action has been chosen not break the supersymmetry on the brane. It is intresting that even in case of $k = 0$, the quantum effect is included in (56) through the parameter C (coefficient of dilatonic term in conformal anomaly). In the classical case, where $C = 0$, the value of the scalar field ϕ_0 is a free parameter. Quantum effects suggest the way for dynamical determination of brane dilaton.

When $k \neq 0$, by substituting the solution in (47) into (53) and (54), one finds

$$\frac{p_1 y_0^{\frac{3}{2}}}{4\pi G}\sqrt{\frac{k}{3-2p_1^2}}\left(1 - \frac{6}{\sqrt{8p_1^2-3}}\right) = 6C\phi_0 \tag{57}$$

$$\frac{3y_0^{\frac{3}{2}}}{16\pi G}\sqrt{\frac{k}{3-2p_1^2}}\left(1 - \frac{2p_1^2}{\sqrt{8p_1^2-3}}\right) = -8b' \ . \tag{58}$$

Since $b' < 0$, Eqs.(57) and (58) have non-trivial solutions for ϕ_0 and y_0 if

$$p_1 > \frac{3}{8} \ , \quad \frac{k}{3-2p_1^2} > 0 \quad \text{and} \quad 1 - \frac{2p_1^2}{\sqrt{8p_1^2-3}} > 0 \ , \tag{59}$$

The last condition in (59) can be rewritten as

$$\frac{1}{2} < p_1^2 < \frac{3}{2} \ . \tag{60}$$

When $k = 3$, where the brane is 4d sphere (de Sitter space when we Wick-rotate the brane metric to Lorentzian signature), we have

$$\frac{3}{2} > p_1^2 > \frac{7}{8} \ . \tag{61}$$

On the other hand, if $k=-3$, where brane is 4d hyperboloid (anti-de Sitter after the Wick-rotation), there is no solution since the second condition in (59) conflicts with (60). Note that de Sitter brane ($k>0$) solution does not exist on the classical level but the solution appeared after inclusion of the quantum effects of brane matter in accordance with AdS/CFT.

If Eq.(61) is satisfied, Eqs.(57) and (58) can be explicitly solved with respect to y_0 and ϕ_0. This situation is very different from the non-supersymmetric case in [7], where S_1 was chosen as in (24). In second ref. from [7], it was very difficult to solve the equations corresponding to (57) and (58), explicitly. This indicates that supersymmetry simplifies the situation and the approach we adopt is right way to construct supersymmetric new brane world. Moreover, quantum effects may give the natural mechanism for SUSY breaking.

If one writes $y_0 = R_b^2$, R_b corresponds to the radius of the sphere ($k=3$). Since $b' \propto N^2$ in (29) and (30) for large N, from Eq.(58), one gets

$$R_b \propto \left(GN^2\right)^{\frac{1}{3}} . \tag{62}$$

Note that $\frac{1}{R_b}$ corresponds to the expanding rate of the de Sitter Universe after the Wick-rotation. Therefore for the large quantum effect (i.e., when N is large), the rate becomes small.

Note that supersymmetry on the de Sitter brane should be broken since such spacetime is not supersymmetric background. On the classical level there is no de Sitter brane solution but only flat brane solution with $k=0$. This would tell that supersymmetry of the whole system does not break down on the classical level, even if brane is introduced. Quantum effects induced by the trace anomaly of brane matter (in accordance with AdS/CFT) could break the supersymmetry of the system including the brane and allow the de Sitter brane solution.

4 Gravity perturbations

It is known that brane gravity trapping occurs on curved brane in a different way than on flat brane. For example, in refs.[12, 13](for dilaton presence, see last ref. in [7]) , the AdS_4 branes in AdS_5 were discussed and the existence of the massive normalizable mode of graviton was found. In these papers, the tensions of the branes are free parameters but in the case treated in the present paper the tension is dynamically determined. As brane solutions are found in the previous section when the brane is flat or de Sitter space, it is reasonable to consider perturbation around the solution.

Let us regard the brane as an object with a tension $\tilde{U}(\phi)$ and assume the brane can be effectively described by the following action, as in (6) (for simplicity, we only consider the brane corresponding to $i = 2$, or the limit $z_1 \to -\infty$):

$$S_{\rm brane} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g_{(4)}} \tilde{U}(\phi) \ . \tag{63}$$

Then using the Einstein equation as in (10), one finds

$$A'' + 4(A')^2 = \frac{V}{3} - \frac{\tilde{U}(\phi)}{6} \delta(z - z_0) \ . \tag{64}$$

Here we assume that there is a brane at $z = z_0 (= z_2)$. Then at $z = z_0$

$$A'|_{z=z_0} = -\frac{\tilde{U}(\phi)}{6} \ . \tag{65}$$

Comparing (65) with (38) etc. one gets, when $k \neq 0$

$$\tilde{U}(\phi) = -\frac{3}{l} W(\phi_0) + \frac{48\pi G b'}{R_b^4} \ . \tag{66}$$

and when $k = 0$

$$\tilde{U}(\phi) = -\frac{3}{l} W(\phi_0) \ . \tag{67}$$

Note that tension becomes R_b dependent due to the quantum correction when $k \neq 0$, as $b' \sim \mathcal{O}(N^2)$ and $R_b^4 \sim \mathcal{O}\left(N^{\frac{8}{3}}\right)$ from (62), the tension depends on N as $\tilde{U}(\phi) + \frac{3}{l} W(\phi_0) \sim \mathcal{O}\left(N^{-\frac{2}{3}}\right)$. One can understand that r.h.s. in (67) and the first term in r.h.s. in (66) are determined from the supersymmetry.

We now consider the perturbation by assuming the metric in the following form:

$$ds^2 = \mathrm{e}^{2\hat{A}(\zeta)} \left(d\zeta^2 + \left(\hat{g}_{\mu\nu} + \mathrm{e}^{-\frac{3}{2}\hat{A}(\zeta)} h_{\mu\nu} \right) dx^\mu dx^\nu \right) \ . \tag{68}$$

The following gauge conditions are chosen

$$h^\mu{}_\mu = 0 \ , \quad \nabla^\mu h_{\mu\nu} = 0 \ . \tag{69}$$

Then one obtains the following equation

$$\left(-\partial_\zeta^2 + \frac{9}{4}\left(\partial_\zeta \hat{A}\right)^2 + \frac{3}{2}\partial_\zeta^2 \hat{A}\right) h_{\mu\nu} = m^2 h_{\mu\nu} \tag{70}$$

Here m^2 corresponds to the mass of the graviton on the brane

$$\left(\hat{\Box} + \frac{1}{R_b^2}\right) h_{\mu\nu} = m^2 h_{\mu\nu} \ . \tag{71}$$

for $k > 0$ and

$$\hat{\Box} h_{\mu\nu} = m^2 h_{\mu\nu} \tag{72}$$

for $k = 0$. Here $\hat{\Box}$ is 4 dimensional d'Alembertian constructed on $\hat{g}_{\mu\nu}$. Since

$$\pm \mathrm{e}^A d\zeta = dz = \sqrt{f} dy \ , \quad \mathrm{e}^A = \frac{\sqrt{y}}{l} \ , \tag{73}$$

one finds

$$\pm\zeta = \int dy \sqrt{\frac{f(y)}{y}} \ . \tag{74}$$

If we choose $\zeta = 0$ when $y = y_0$, Eq.(74) for the solution in (47) or (50) gives

$$|\zeta| = -\frac{1}{q}\ln y + \frac{1}{q}\ln y_0 \ . \tag{75}$$

Here we assume q is defined by (49) even if $k > 0$. Since only the square of q is defined in (49), we can choose q to be positive.

Note that brane separates two bulk regions corresponding to $\zeta < 0$ and $\zeta > 0$, respectively. Since y takes the value in $[0, y_0]$, ζ takes the value in $[-\infty, \infty]$. Since $A = \frac{1}{2}\ln y$, from (70) one gets

$$\left(-\partial_\zeta^2 + \frac{9q^2}{4} - 3q\delta(\zeta)\right) h_{\mu\nu} = m^2 h_{\mu\nu} \tag{76}$$

Zero mode solution with m^2 of (76) is given by

$$h_{\mu\nu} = h^{(0)}_{\mu\nu} \mathrm{e}^{-\frac{3q}{2}|\zeta|} \ . \tag{77}$$

Here $h^{(0)}_{\mu\nu}$ is a constant. Any other normalizable solution does not exist. When

$$m^2 > \frac{9}{4} q^2 \ , \tag{78}$$

there are non-normalizable solutions given by

$$h_{\mu\nu} = a_{\mu\nu} \cos\left(|\zeta| \sqrt{m^2 - \frac{9}{4} q^2} \right) + b_{\mu\nu} \sin\left(|\zeta| \sqrt{m^2 - \frac{9}{4} q^2} \right) . \qquad (79)$$

The coefficients $a_{\mu\nu}$ and $b_{\mu\nu}$ are constants of the integration and they are determined to satisfy the boundary condition, which comes from the δ-function potential in (76),

$$\left. \frac{\partial_\zeta h_{\mu\nu}}{h_{\mu\nu}} \right|_{\zeta \to 0+} = -\frac{3}{2} q \ . \qquad (80)$$

Note that zero mode solution (77) satisfies this boundary condition (80). By using (80), we can determine the coefficients $a_{\mu\nu}$ and $b_{\mu\nu}$ for non-normalizable solutions as follows:

$$a_{\mu\nu} = -\frac{2b_{\mu\nu}}{3q} \sqrt{m^2 - \frac{9}{4} q^2} \ . \qquad (81)$$

It might be interesting that there is the minimum (78) in the mass of non-normalizable mode. This situation is different from the original Randall-Sundrum model [2]. Although the de Sitter brane appears when we include the quantum correction, the minimum itself does not depend on the parameter of the quantum correction b' or N.

Since there is only one normalizable solution corresponding to zero mode (77) and other solutions (79) are non-normalizable, the gravity should be localized on the brane and the leading long range potential between two massive sources on the brane should obey the Coulomb law, i.e., $\mathcal{O}\left(r^{-1}\right)$. Here r is the distance between the above two massive sources. Furthermore the existence of the the minimum (78) in the mass of non-normalizable mode indicates that the correction to the Coulomb law should be small.

5 Discussion

In summary, the attempt to supersymmetrize the quantum-induced dilatonic New Brane World motivated by AdS/CFT is done. It is shown that for number of superpotentials one can construct flat SUSY dilatonic brane-world or de Sitter dilatonic brane-world where SUSY is broken by quantum effects. The crucial role in the creation of de Sitter brane Universe (not Anti-de

Sitter one!) is in account of quantum effects which produce the effective brane tension. The analysis of graviton perturbations for such brane-worlds shows that gravity trapping on the brane occurs.

It would be interesting to investigate the scenario of such sort in situation when not only scalar-gravitational background is non-trivial as in present work but also when other superpartners have non-trivial background. Clearly, it is not so easy as the quantum effective action is getting much more complicated in such case. From another side, in the present discussion the only brane quantum effects are taken into account. One possibility could be in account of bulk quantum effects (bulk Casimir effect [14] in simplest version) in the construction of supersymmetric brane-worlds.

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